Applications of higher order Seifert–van Kampen Theorems for structured spaces

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Abstract

The purpose of this note is to give clear references to the many applications of higher order Seifert–van Kampen theorems, in terms of specific calculations in homotopy theory and related areas; such theorems refer to all involve spaces with structure, either a filtration, or an $n$-cube of spaces. Numbers [xx] refer to my publication list http://pages.bangor.ac.uk/~mas010/publicfull.htm. Numbers [[xx]] at the end of an item refer to the number of citations given on MathSciNet.

Applications of a 2-dimensional Seifert van Kampen Theorem for pairs of spaces, with values in crossed modules


V. G. Bardakov, R. Mikhailov, V. V. Vershinin, J. Wu, ‘Brunnian Braids on Surfaces’, arXiv:0909.3387 (This refers to [51] but really gives an application of [43].)


Applications of an $n$-dimensional Seifert van Kampen Theorem for filtered spaces, with values in crossed complexes


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This paper, which generalises [25] to all dimensions, gives a new approach to the border between homotopy and homology by working with filtered spaces, and, using homotopical constructions, gives what we now call a Higher Homotopy Seifert van Kampen Theorem. From this, without setting up singular homology, or using simplicial approximation, one proves:

A. The Brouwer Degree Theorem (the $n$-sphere $S^n$ is $(n-1)$-connected and the homotopy classes of maps of $S^n$ to itself are classified by an integer called the degree of the map);

B. The Relative Hurewicz Theorem, which is seen here as describing the morphism

$$\pi_n(X, A, x) \rightarrow \pi_n(X \cup CA, CA, x) \overset{\cong}{\rightarrow} \pi_n(X \cup CA, x)$$

when $(X, A)$ is $(n - 1)$-connected, and so does not require the usual involvement of homology groups.

The following book contains a comprehensive survey of much of the above, and more, including in Part I results in dimensions 1 and 2 which should be seen as an aspect of low dimensional topology.


Applications of a higher dimensional Seifert–van Kampen Theorem for $n$-cubes of spaces, with values in crossed squares, cat$^n$-groups, and crossed $n$-cubes of groups


This paper introduced a nonabelian tensor product of groups which act on each other. A current bibliography on this subject with 120 items is at [http://pages.bangor.ac.uk/~mas010/nonabtens.html](http://pages.bangor.ac.uk/~mas010/nonabtens.html).


Ellis, G. J. ‘The group $K_2(A; I_1, \cdots, I_n)$ and related computations’. *J. Algebra* 112 (2) (1988) 271–289.

