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Brown, Ronald; **Higgins, Philip J.**; **Sivera, Rafael**
Nonabelian algebraic topology. Filtered spaces, crossed complexes, cubical homotopy groupoids. With contributions by Christopher D. Wensley and Sergei V. Soloviev. (English)
EMS Tracts in Mathematics 15. Zürich: European Mathematical Society (EMS) (ISBN 978-3-03719-083-8/hbk). xxxv, 668 p. (2011).

The development of algebraic topology has tended to favour abelian valued invariants. Homology and cohomology are obvious examples. Chain complexes and spectra are central to much of that theory and when non-abelian groups, such as the fundamental group of a space, do appear they are quickly put aside by passing to the group ring and considering modules over them. All this does destroy (or obscure) information. It is possible to work with fundamental groups and with other non-abelian invariants, such as the associated crossed modules of CW-structures, in a less drastic way, retaining the `non-abelianness' a bit longer, and thus retaining also more of the homotopical information in the process. This idea was pioneered by Henry Whitehead in his work in the late 1940s and 1950s, and was pushed forward by Brown and Higgins in the 1980s at which time was developed a large toolkit of methods relating to generalisations of the Seifert-van Kampen theorem and thus to the calculation of these non-abelian invariants.

This book is one of the first to put together, in one place, this non-abelian approach to homotopy theory. The approach is via the groupoid based Seifert-van Kampen theorem which has clearer geometric content than the group based one, and then suggests ways of approaching higher dimensional higher groupoid-based analogues of it. One of the key ideas or themes running through the book is thus that algebraic colimit arguments usually involving higher dimensional groupoids, reflect the `geometry' of the topological situation better than do, for instance, arguments involving exact sequences.

The majority of the results in the book have appeared in papers by Brown and Higgins. Here they are put together in one place, and with consistent notational conventions, good cross references, introductory material and applications. The book is designed to be readable by a graduate student who has taken courses in topology, some homotopy theory and an introductory course in category theory. (Additional material giving more categorical background is included in some useful appendices.)

After the usual sections on prerequisites, there is an excellent introduction laying out some of the main themes and ideas that will be met later on. This is followed by Part I, which looks at the 1- and 2-dimensional results and the key step of going from a group based theory to one involving double groupoids and crossed modules. Part II then looks at crossed complexes, the higher dimensional Seifert-van Kampen theorem and its consequences and applications to various contexts. Part III examines the necessary parallel theory of cubical ω-groupoids and how that provides the necessary underpinning for the results of Part II. This is followed by the Appendices, a large bibliography and useful indices of notation and terminology.

Reviewer: Timothy Porter (Bangor)

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