We are now able to state our enriched version of the result of Elmendorf, [22]. Other authors have obtained variants of this, notably Seymour, [32], and Dwyer and Kan, [20]. The following theorem is one half of [17, Theorem 3.11(i)]. In this theorem, by G-complex we mean G-CW-complex.

Theorem 2.2 ([17]). *The above pair of $\mathcal{S}$-functors*

$$R : G\text{-}\mathcal{T}op \to \mathcal{S}^{OrG^{\mathrm{op}}}, \qquad c : \mathcal{S}^{OrG^{\mathrm{op}}} \to G\text{-}\mathcal{T}op$$

has the properties that if Y is a G-complex, and T is a OrG^{op}-diagram taking Kan values, then there is a homotopy equivalence of Kan simplicial sets

$$G\text{-}\underline{\mathcal{T}op}(Y, c(T)) \simeq Coh\,\underline{\mathcal{S}}(R(Y), T).$$

In order to prove that certain key maps are homotopy equivalences, we need the alternative construction of the simplicial set of homotopy coherent transformations using ends rather than coends. Since we will form indexed limits, we require the receiving $\mathcal{S}$-category $\mathcal{C}$ to be complete. Then $\mathcal{C}$ will be *cotensored*, which means that if K is a simplicial set and C an object of $\mathcal{C}$, there is an object which we will denote by $\bar{C}(K, C)$ such that there is a natural isomorphism

$$\underline{\mathcal{S}}(K, \underline{\mathcal{C}}(C', C)) \cong \underline{\mathcal{C}}(C', \bar{C}(K, C)).$$

Suppose

$$Q : \mathcal{A}^{\mathrm{op}} \times \mathcal{A} \to \mathcal{C}.$$

We can now define the *homotopy coherent end* of Q by

$$\oint_A Q(A, A) = \int_{A, A'} \bar{C}(X(A, A'), Q(A, A'))$$

(cf. Cordier and Porter, [18]).

Example 2.3. Suppose $F, G : \mathcal{A} \to \mathcal{C}$ are two $\mathcal{S}$-functors, and set $Q(A, A') = \underline{\mathcal{C}}(FA, GA')$. Then $\oint_A Q(A, A)$ can be interpreted as the simplicial set of homotopy coherent transformations from F to G. This will be denoted by $Coh(\mathcal{A}, \underline{\mathcal{C}})(F, G)$ or more simply by $Coh\,\underline{\mathcal{C}}(F, G)$ if the codomain is the important information to remember whilst the domain is fixed, or just by $Coh(F, G)$ if there is no danger of confusion.

Given an $\mathcal{S}$-functor $Q : \mathcal{A}^{\mathrm{op}} \times \mathcal{A} \to \mathcal{C}$, we can construct a cosimplicial object in $\mathcal{C}$, denoted $Y(Q) : \Delta \to \mathcal{C}$, by

$$Y(Q)^n = \prod \{Q(A_0, A_n) : u \in (Ner\,\mathcal{A})_n, u = (A_0 \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_n} A_n)\}. \tag{10}$$

The coface and codegeneracy maps are given by formulae analogous to those of the 'cosimplicial replacement' construction of Bousfield and Kan, [7], and are given in detail in [18]. As is now standard, the Bousfield–Kan homotopy limit of a diagram of simplicial sets can be given as a 'total complex' of a cosimplicial simplicial set constructed from the given data. If Y is a cosimplicial simplicial set, dijk and Svensson [31] on equivariant 2-types, but with results on function spaces and not just homotopy classes of maps.

It is also possible to formulate a result for equivariant homotopy types with non-trivial homotopy groups only in dimensions 1 and n. Such a homotopy type is modelled by a crossed complex C which has precisely these groups as its homology.

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