Category theory, higher dimensional algebra, groupoid atlases

Prospective descriptive tools in theoretical neuroscience

Ronnie Brown

Asklöster

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A few possibilities!

Challenge: The cricket cercal system

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G.A. Jacobs, J.P. Miller, Z.Aldworth, 'Computational mechanosensory processing in the cricket', J. Exp. Biol., 211 (2008) 1819-1828.


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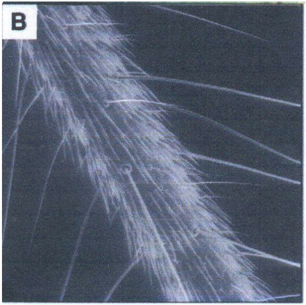

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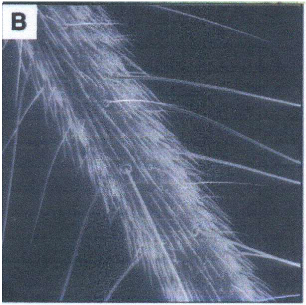
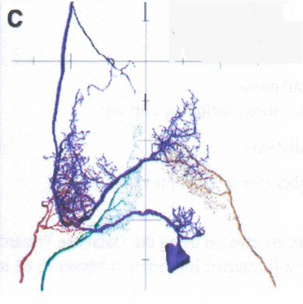


B. Scanning electron microscope close-up of a segment of the cercus. The cercus is approximately 1 cm in length.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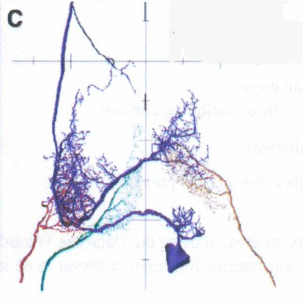
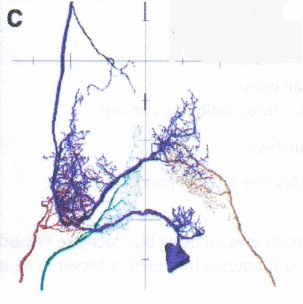


C. Computer reconstructions of a primary sensory Interneuron (blue) and three primary sensory afferents (red, light blue and brown) in their correct anatomical relationships. Scale: 40 μm between tick marks on the scale bars.



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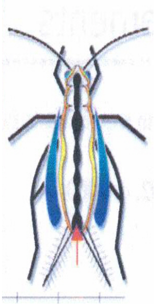

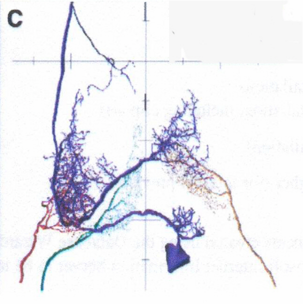

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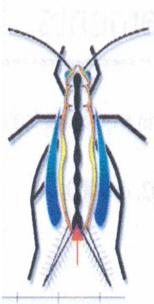

Cartoon of a cut-away view of the cricket nervous system. The terminal abdominal ganglion, where the sensory neurons and interneurons are located, is indicated with a red arrow.

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(c) coding of the spectral composition of dynamic air currents via the relative activity levels of different interneurons having different frequency sensitivity bands, and

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Also you want that the final received email is independent of all the choices that have been made.

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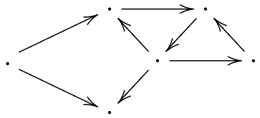

'Functional controls': cocone *with base D and vertex an object C.*

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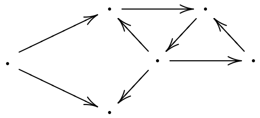
$D =$ 

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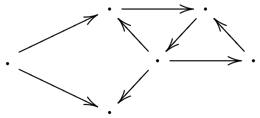
$D =$ 

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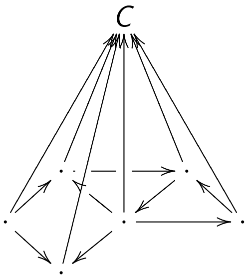

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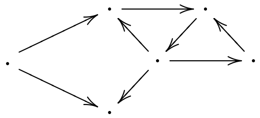

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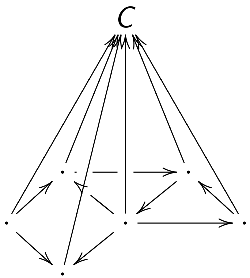
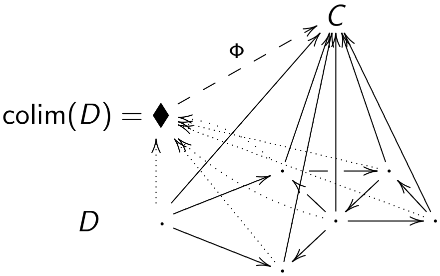
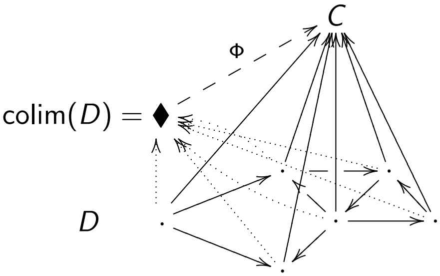
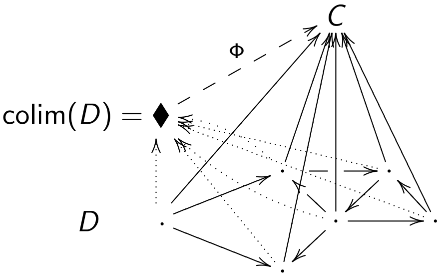


such that each of the triangular faces of this cocone is commutative.


C
Φ
$\mathrm{colim}(D) = \blacklozenge$
D



The output will be an object $\mathrm{colim}(D)$ in our category C defined by a special *colimit cocone*



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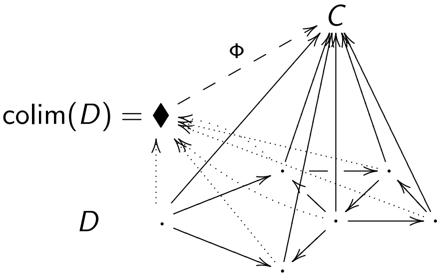

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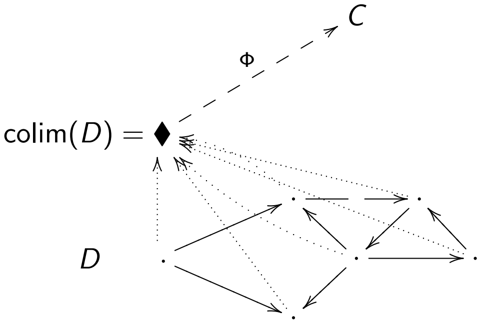
Again, all triangular faces of the combined picture are commutative.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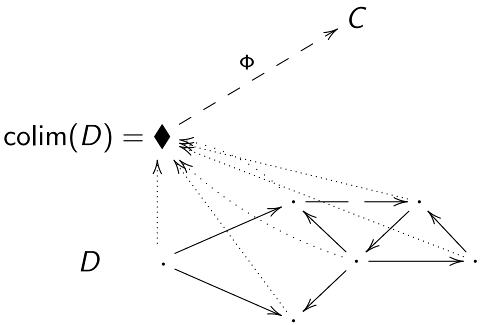
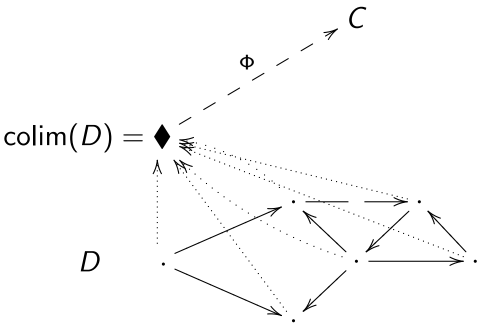


The commutativity condition on the cocone in essence forces interaction in the colimit of different parts of the diagram D.



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Memory Evolutive Systems

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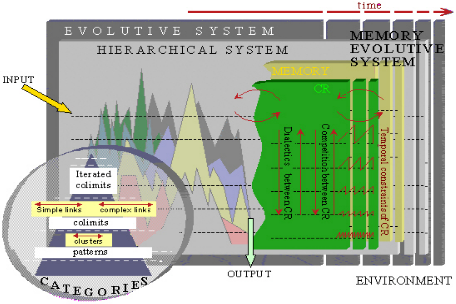

"The theory of Memory Evolutive Systems represents a mathematical model for natural open self-organizing systems, such as biological, sociological or neural systems. In these systems, the dynamics are modulated by the cooperative and/or competitive interactions between the global system and a net of internal Centers of Regulation (CR) with a differential access to a central hierarchical Memory." (from the advertisement for the book).

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DARPA Mathematical Challenges Defense Sciences Office;
DARPA-BAA 08-65 September 26, 2008
Mathematical Challenge One: The Mathematics of the Brain
Develop a mathematical theory to build a functional model of the brain that is mathematically consistent and predictive rather than merely biologically inspired.

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Higher dimensional category theory gives a rich source of algebra and of concepts not yet integrated with analysis but with many applications to differential geometry.

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Groupoid atlases: patching groupoids

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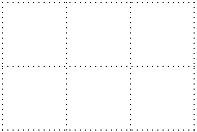

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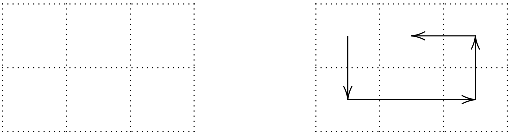
Groupoid atlases: patching groupoids

The objects of a groupoid give a spatial component to group theory enabling a patching idea and a possibility of transferring from one patch to another.

Origins in algebraic K-theory: $GL(n, R)$ and its family of elementary subgroups, with actions of the big group on cosets.

Need for a theory to accommodate for example:



set with two equivalence relations

path through equivalence classes

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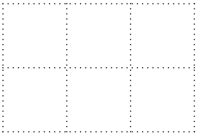
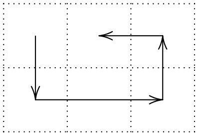


set with two equivalence relations



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Similarly, a set with several group actions.

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Thus the fact that a groupoid has objects enables a patching idea and a possibility of transferring from one patch to another.

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Can we model in computers the way mathematicians actually work?

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These are characteristics of symbolic communication and so should be built into computer programs modelling mathematics, and into models of systems communication. The cricket example shows there is a long way to go!