Bull. London Math. Soc. 39 (2007) 867–872

BOOK REVIEWS

TOPOLOGY AND GROUPOIDS

By RONALD BROWN: 512 pp., US$23.99, ISBN 1-4196-2722-8
(www.groupoids.uk, Deganwy, UK, 2006; published by BookSurge LLC,
Charleston, SC, USA, 2006).

doi:10.1112/blms/bdm072
Published online 23 August 2007

This is the third version of a book first published by McGraw-Hill in 1968 as *Elements of modern topology* [1], and reissued by Ellis Horwood in 1988 under the title *Topology: a geometric account of general topology, homotopy types and the fundamental groupoid* [3].

The changes in the title reflect the developments in the author's research in topology, which he has pursued with energy and dedication over the years. Each of the new versions has been carefully revised, updated and expanded. During his work on the original book, the author had already realized the potential for applications of the notion of groupoids in algebraic and geometric topology, in particular in homotopy theory. He quickly gained the conviction that the standard group theory in connection with calculations of homotopy groups — in particular the van Kampen theorem for calculating the fundamental group — had a generalization to higher dimensions, when group theory is rephrased as groupoid theory.

Ronnie Brown has been a tireless proponent of the role of groupoids in mathematics, with the survey paper [2] marking the end of the beginnings. From his base at the University of Wales at Bangor, Brown has succeeded in building up a worldwide group of researchers working in the area, including many of his former students. There is still some way to go before the theory of groupoids occupies the place in the teaching of algebraic topology that Ronnie Brown would like to see, and that it might deserve. Hence it is most valuable and appropriate that an expert in the subject should work on presenting the details in the theory as clearly as possible. He cares for exposition, and he has an eminent and convincing style of writing, of which you get a good impression by reading his book.

The original book was intended as a textbook on point set and algebraic topology at the undergraduate and the beginning graduate level. The present version still achieves this goal, but is now supplemented with material on the more advanced parts of the theory of groupoids, which belongs to later stages of doctoral studies.

The point set topology of the book covers the needs of students in algebraic and geometric topology in a most adequate way, with emphasis on spaces defined by identification, adjunction spaces and cell complexes. For the needs of students of analysis, the coverage of point set topology is less adequate, putting little weight on various notions of convergence.

The main mathematical structure introduced and studied in the book is the fundamental groupoid of a topological space. Since the terminology of category theory prevails, the *fundamental groupoid* πX of a topological space X is viewed as a category with the points of X as the objects and the homotopy classes (relative to end points) of continuous paths with the same parameter interval as morphisms. Composition of morphisms — if defined — is by concatenation of paths.

A large part of the book is dedicated to methods used for computation of the fundamental groupoid, with the van Kampen theorem as an especially useful tool. Two of the final chapters of the book are devoted to 'covering spaces and covering groupoids' and 'orbit spaces and orbit groupoids', respectively.

At several places in the book — in particular, in the last chapter of the book, entitled *Conclusion* — the author gives us charming insights into his struggles with groupoids, his views on choosing research problems, the 'life of a mathematician', and the difficulty in recognising a change of paradigm in mathematics 'even for a person involved in the change!' (p. 437).

The book is available in an electronic version on a CD Rom, with full hyperref and some colour. The CD Rom is sold for £7.

I think there is much more to groupoids than is presently realized by topologists. Ronnie Brown is a major contributor to the field, and a master of the subject. His book is well written, with care for details, and he makes an excellent case for groupoids.

References

1. R. Brown, *Elements of modern topology*, European Mathematics Series (McGraw-Hill, 1968).
2. R. Brown, 'From groups to groupoids: a brief survey' *Bull. London Math. Soc.* 19 (1987) 113–134.
3. R. Brown, *Topology: a geometric account of general topology, homotopy types and the fundamental groupoid*, updated and expanded edn, Ellis Horwood Series in Mathematics and its Applications (Ellis Horwood, Chichester, UK, 1988).

Technical University of Denmark — Vagn Lundsgaard Hansen

A GUIDE TO CLASSICAL AND MODERN MODEL THEORY
(Trends in Logic — Studia Logica Library 19)

By Annalisa Marcja and Carlo Toffalori: 369 pp.,
£77.00 (US$115.00, 120.00 euro), isbn 1-4020-1330-2
(Kluwer Academic Publishers, Dordrecht, 2003).

doi:10.1112/blms/bdm071
Published online 23 August 2007

Model theory, while being part of mathematical logic, bears similarities to category theory and abstract algebra. It is a subject of enormous generality and abstraction, with its own internal dynamic, agendas and problems, but it comes to life when applied to more concrete mathematical contexts and objects. The basic objects of model theory, at least from the semantic point of view, are so-called structures (or models). On the face of it, a structure M is something quite naive: simply a set X (the universe of M) equipped with a distinguished collection $\mathcal{F}$ of pointsets, subsets of $X, X \times X, X \times X \times X, \ldots$. From the model-theoretic point of view, however, the structure $M = (X, \mathcal{F})$ comes with additional baggage, namely:

(a) $\mathrm{Def}(M)$, the category of definable sets in M — that is, the sets obtained from the family $\mathcal{F}$ by the operations of finite Boolean combinations and projections, and
(b) the family of all 'nonstandard' versions of M, otherwise called elementary extensions of M.

For example, if we take X to be the set of complex numbers, and $\mathcal{F}$ to consist of the diagonal as well as the graphs of addition and multiplication, then $\mathrm{Def}(M)$ will be essentially the category of complex algebraic varieties, and the elementary extensions of M correspond to algebraically closed fields containing the complexes.